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Performance Evaluation of CFO in Single Carrier-FDMA

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ABSTRACT: Single-carrier frequency division multiple access (SC-FDMA) a modified form of Orthogonal FDMA, is a promising technique for high data rate and low PAPR for uplink communications in future cellular systems. However, like OFDM, SC-FDMA also suffers from carrier frequency offset (CFO) and this is a major problem especially for uplink communication, where more than one CFOs exist. There are two kinds of effects that occur due to CFO; inter carrier interference (ICI), which occurs between a user's own subcarriers and multiple access interference (MAI) which occurs between multiple users' subcarriers. In this paper, we provide some basic details about SC-FDMA and we also find the BER and power spectral density (PSD) and also compare PAPR of OFDM and SC-FDMA at different FFT points with BPSK and QPSK modulation schemes. We also analyze two problems Inter symbol interference (ISI) and Carrier Frequency Offsets (CFOs) which are present in SC-FDMA and use methods to remove or minimize them. Here, we use a suppression method to overcome the multiple CFOs estimation and suppression in LTE uplink problem in which null subcarriers which are present in LTE uplink standard is used to measure the energy of signal in null subcarriers and use CFO value which provides minimum energy during suppression.

KEYWORDS: SC-FDMA, LTE, CFO, ICI, MAI, CFO synchronization, BER, PSD.

I. INTRODUCTION

3GPP Long Term Evolution (LTE) is the telecommunication network of the next generation, following 3G. The main advantages of this new technology are high data rate, low latency and packet optimized radio access technology. The LTE specification provides uplink peak rates of at least 50 Mbps in 20 MHz system bandwidth [1, 2, 4]. For the realization of LTE Uplink Single Carrier Frequency Division Multiple Access (SC-FDMA) transmission is used. The reason of choosing this technology is that SC-FDMA has sufficiently low Peak-to-Average Power ratio (PAPR) of signals in comparison with Orthogonal Frequency Division Multiple Access (OFDMA) transmission. It results in significantly lower power consumption in the user equipment (UE).

Single-carrier FDMA (SC-FDMA) is a FDMA scheme. Like other multiple access schemes (TDMA, FDMA, CDMA, OFDMA), it deals with the assignment of multiple users to a shared communication resource. SC-FDMA can be interpreted as a linearly pre-coded OFDMA scheme, in the sense that it has an additional DFT processing preceding the conventional OFDMA processing. SC-FDMA has drawn great attention as an attractive alternative to OFDMA, especially in the uplink communications where lower peak-to-average power ratio (PAPR) greatly benefits the mobile terminal in terms of transmit power efficiency and reduced cost of the power amplifier. It has been adopted as the uplink multiple access scheme in 3GPP long term evolution (LTE), or Evolved UTRA (E-UTRA) [3].

On the other hand, like other OFDM based systems, SC-FDMA is vulnerable to CFO, which is mainly caused by Doppler shift and oscillator mismatch between the transmitter and the receiver. CFO disturbs the orthogonality among subcarriers, thus decreases the system performance severely. In uplink communications, multiple signals sent from multiple users are affected by different CFO values. After passing from these channels, these signals are combined at the receiver. This combination results in MAI. In order to obtain a reliable system performance, efficient CFO synchronization is very important [7].

As different users have different CFOs, synchronization of CFO is harder in uplink than in downlink scenarios. There have been CFO correction methods for downlink communications however these methods are applicable for only single-user systems and gives inferior performance in uplink communications [5]. These kinds of methods are also feasible for SC-FDMA systems, since they are designed for uplink communications and an OFDM based technique (OFDMA) like SC-FDMA [6].

II. SYSTEM MODEL

Single-carrier FDMA (SC-FDMA) is a frequency-division multiple access scheme. SC-FDMA can be interpreted as a linearly precoded OFDMA scheme, in the sense that it has an additional DFT processing preceding the conventional OFDMA processing. FDMA has drawn great attention as an attractive alternative to OFDMA, especially in the uplink communications where lower peak-to-average power ratio (PAPR) greatly benefits the mobile terminal in terms of transmit power efficiency and reduced cost of the power amplifier [3].

In addition to the low-PAPR ('single-carrier' property) inherent in SC-FDMA, it also has some other desired properties for a transmission scheme. SC-FDMA allows for the possibility for low-complexity but high-quality equalization in the frequency domain and it is also possible to have flexible bandwidth assignments with SC-FDMA.

Another benefit of SC-FDMA is the so-called "built-in" frequency diversity. Because SC-FDMA spread the information of one symbol through all the available subcarriers, so in case losing partial information on one (or even more) subcarriers due to deep fading does not necessarily lead to losing the information modulated in the symbol [3].

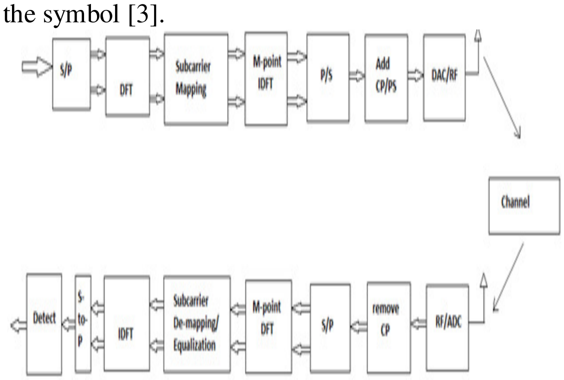

Fig.1. Block diagram of SC-FDMA.

The main difference between OFDM and SC-FDMA transmitter is the DFT mapper which is shown in fig. 1. DFT transforms these symbols in time domain into frequency domain. Mathematically:

$$D_i^k = \frac{1}{M}\sum_{m=0}^{M_k-1} d_m^k . e^{-j2\pi mi/M_k} \qquad \ldots(1)$$

Where,

M_k is the subcarrier number of S_k, where S_k is the sub-channel group assigned to K^{th} user. M is the number of sub-carriers. K is the active users.

The frequency domain samples are then mapped to a subset of M subcarriers as

$$X^k = [X_0^k, X_1^k, X_2^k, \ldots, X_{M-1}^k]'$$

Similar to OFDM, an M-point IDFT is used to generate the time-domain samples of these subcarriers, which is followed by cyclic prefix, parallel to serial converter, DAC and RF subsystems. Mathematically equation of IDFT is:

$$s^k(n) = \sum_{l=0}^{M-1} X_l^k . e^{j2\pi nl/M} = \sum_{l=0}^{M-1} X_l^k . p_{n,l} \quad \ldots(2)$$

DFT output of the data symbols is mapped to a subset of subcarriers, a process called subcarrier mapping. The subcarrier mapping assigns DFT output complex values as the amplitudes of some of the selected subcarriers. Subcarrier mapping can be classified into two types: *localized mapping* and *distributed mapping*.

A. Subcarrier Mapping

In *localized mapping*, the DFT outputs are mapped to a subset of consecutive sub-carriers thereby confining them to only a fraction of the system bandwidth as shown in fig. 2. In *distributed mapping*, the DFT outputs of the input data are assigned to subcarriers over the entire bandwidth non-continuously, resulting in zero amplitude for the remaining subcarriers. A special case of *distributed SC-FDMA* is called *interleaved SC-FDMA*, where the occupied subcarriers are equally spaced over the entire bandwidth as shown in fig. 3.

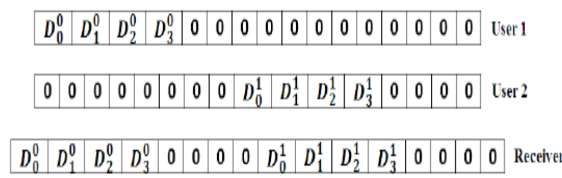

Fig. 2. An exemplary LFDMA subcarrier mapping with sub-bands.

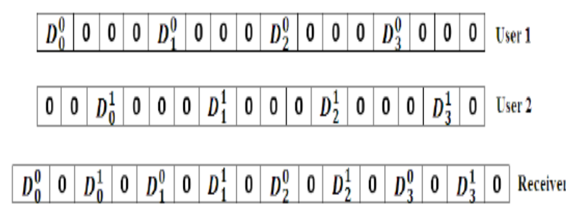

Fig. 3. An exemplary IFDMA subcarrier mapping with sub-bands.

IFDMA is a special case of SC-FDMA and it is very efficient in that the transmitter can modulate the signal strictly in the time domain without the use of DFT and IDFT. Then inverse-FFT (IFFT) is used to transform the modulated subcarriers in frequency domain to time domain samples.

Cyclic prefix (CP) longer than the channel response length is needed to convert linear convolution to circular convolution.

The Cyclic Prefix or Guard Interval is a periodic extension of the last part of symbol that is added to the front of the symbol in the transmitter, and is removed at the receiver before demodulation. The CP has two purposes here; the first is that it is use as a guard interval to eliminate the Inter Symbol Interference (ISI) from the previous symbol. The second is that prefixing the symbols with repetition of the end makes symbol periodic and linear convolution with channel will changes to circular convolution. In frequency domain its equivalent to point-wise multiplication of symbols to channel frequency response. To ease equalization, the length of CP should be minimum equal to maximum delay in the channel or in other words equal to the delay spread of the channel. Before modulating the signal with high frequency to transmit there is a pulse shaping filter that will shape the signal to get the desired spectrum.

At the receiver side, do exactly the inverse of what done at transmitter. Demodulate the signal to a lower frequency. After that remove cyclic prefix. After removing the CP the receiver transform the received signal into frequency domain with the help of DFT. It then de-maps the sub-carriers and then performs frequency domain equalization. Normally the most common equalizer used is minimum mean square error (MMSE) frequency domain equalizer. The equalized signal is then transformed to time domain by IDFT and detection is done in time domain [3].

Advantages of SC-FDMA are: it having low PAPR, due to which cost of amplifier is reduced and it improves the battery life. It also having low sensitivity to carrier frequency offset. It is less sensitive to non-linear distortion and hence, it allows the use of low cost power amplifier. SC-FDMA having greater robustness against spectra nulls. With these advantages SC-FDMA also having lots of disadvantages. SC-FDMA having higher receiver complexity, additional DFT processing increase mobile station complexity, unlike OFDMA, localized SC-FDMA cannot exploit full advantage of multiuser diversity. Distributed SC-FDMA has not been adopted by 3GPP LTE, because of multiple issues examples: (1) vulnerability to Doppler and frequency offset, (2) pilot design. It has low flexibility in multiplexing uplink control and data channels. With all these disadvantages, SC-FDMA preferred for uplink communication because it having low peak–to-average-power ratio.

Some of the Application of SC-FDMA is: (1) Uplink communication- LTE uses SC-FDMA on the uplink in 4G mobile communication to exploit its PAPR advantages to reduce transmit power back-off in user terminals. (2) Broadband communication to high speed trains**.** (3) The benefits of using SC-FDMA on the downlink are that SC-FDMA can achieve: Significantly better BER at the user terminal compared to OFDMA [11].

III. PROBLEM IN SC-FDMA

A. Inter-Symbol Interference

Inter-symbol interference (ISI) is a form of distortion of a signal in which one symbol interferes with subsequent symbols. ISI is usually caused by multipath propagation or the inherent non-linear frequency response of a channel causing successive symbols to "blur" together. Therefore, to remove this, we use cyclic prefix which is an extension of the last part of symbol to the front of symbol.

B. Carrier Frequency Offset

CFO can destroy orthogonality among sub-carriers and degrades performance of SC-FDMA.

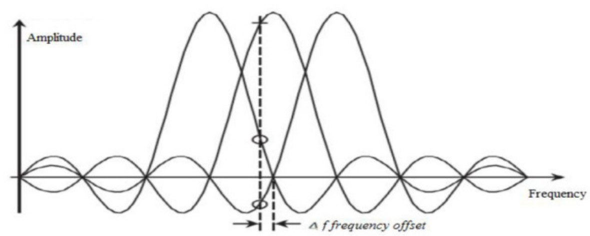

Fig. 4. ICI arises due to frequency offset.

This is a major problem especially for uplink, where more than one CFO exists. CFO synchronization is harder in uplink than in downlink, due to this multiple CFOs. Two kinds of effects that occurs owing to CFOs:

(1) ICI -which occurs between a user's own sub-carriers. (2) MAI –multiple access interference, which occurs between different user sub-carriers. Several methods are used to mitigate effects of CFO, some are here (i) a combined MMSE (minimum mean square error) –FDE (frequency domain equalization) and interference cancellation scheme is proposed [8].

In this scheme, joint FDE with CFO compensation (JFC) is utilized to obtain initial estimation for each user. (ii) Another method is proposed called suppression method to overcome the multiple CFO estimation and suppression in LTE uplink problem [6,9].

In this method, null sub-carriers which are present in LTE uplink standard are used. The main idea is to measure the energy of signal in null subcarriers and use CFO value which provides minimum energy during CFO suppression.

IV. EFFECT OF CFO IN SC-FDMA

Received signal at the base station is:

$$r(n) = \sum_{k=0}^{K-1}[s^k(n) \otimes h^k(n) + v^k(n)] \qquad \ldots(3)$$

Where, $v^k(n)$ is AWGN noise and $h^k(n)$ is the time domain impulse response.

Assuming that the CFO of the K^{th} client is Δf_k, after passing form channel and removing CP, the acquired signal at the base station could be written as:

$$r(n) = \sum_{k=0}^{K-1}\{[s^k(n) \otimes h^k(n)].e^{j2\pi\Delta f_k n} + v^k(n)\} \qquad \ldots(4)$$

At the BS the incoming signal is firstly passed from BPF, to remove MAI. Assuming the symbol time offset impact is disregarded, after removing CP and passing from FFT block, the output on i^{th} subcarrier might be composed as:

$$Y_i^k = \frac{1}{M}\sum_{n=0}^{M-1}[s^k(n) \otimes h^k(n) + v^k(n)].e^{-j2\pi ni/M} \qquad \ldots(5)$$

After FFT block for frequency domain conversion, the signal on the i^{th} subcarrier in presence of offset can be written as:

$$Y_i = \frac{1}{M}\sum_{n=0}^{M-1} r(n).e^{-j2\pi ni/M}$$
$$= \frac{1}{M}\sum_{n=0}^{M-1}\sum_{k=0}^{K-1}\{[s^k(n) \otimes h^k(n)].e^{j2\pi\Delta f_k n}\}.e^{-j2\pi ni/M} + V_i$$
$$= \frac{1}{M}\sum_{n=0}^{M-1}\sum_{k=0}^{K-1} X_i^k H_i^k.e^{j2\pi\Delta f_k n} + \frac{1}{M}\sum_{n=0}^{M-1}\sum_{k=0}^{K-1}\sum_{l=0,l\neq i}^{M-1} X_l^k H_l^k.e^{j2\pi n(l-i)/M}.e^{j2\pi\Delta f_k n} + V_i \qquad \ldots(6)$$

Here, H_i^k is the frequency domain channel response and V_i is the frequency domain AWGN on i^{th} subcarrier.

For each of the subcarrier mapping techniques, a subcarrier can just be allocated to one user. Therefore, only one X_i^k which utilizes the i^{th} subcarrier is the transmitted information of the user, and other K-1 X_i^ks are zero. If this X_i^k belongs to the k^{th} user, Y_i can be modified as:

$$Y_i = X_i^{k'} H_i^{k'} I_0^{k'} + \sum_{l=0,l\neq i}^{M-1} X_l^{k'} H_l^{k'} I_{l-i}^{k'} + \sum_{k=0,k\neq k'}^{K-1}\sum_{l=0,l\neq i}^{M-1} X_l^k H_l^k I_{l-i}^k + V_i \qquad \ldots(7)$$

If $\square_k$ is the normalized CFO value, then:

$$I_L^k = \frac{1}{M}\sum_{n=0}^{M-1} e^{j2\pi n(L+\epsilon_k)/M} \qquad \ldots(8)$$

Is the interference coefficient in eq. (7).

Form (7) it can be clearly seen that the received signal at BS has 4 parts. First one related to the original information transmitted on the i^{th} subcarrier of the k^{th} user. Second one relates to ICI caused by other information of k^{th} user. Third one relates to MAI caused by other's information. And last fourth one relates to AWGN.

By using null subcarrier method, to mitigate the effect of CFO we need two additional blocks at the receiver side of SC-FDMA. First is CFO compensation and second is CFO estimation in fig. 5 lies between FFT block and subcarrier de-mapping block [9].

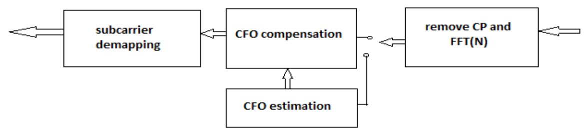

Fig. 5. Block diagram of receiver side of SC-FDMA which includes CFO estimation and compensation.

If the users are stable and frequency offset is caused only by oscillator mismatches, synchronization at only MUs can be enough. Nevertheless, if the users are moving and causing

Doppler shift, a synchronization algorithm is required also at the BS.

The CFO estimation and suppression blocks are primarily based upon the algorithms which are configured for OFDMA [7]. By proper adjustments, this strategy ends up being also feasible for SC-FDMA systems, since it is configured for uplink communications and OFDMA is OFDM-based like SC-FDMA [9].

From fig. 6, firstly, assume that there are two users (k = 0 and k =1) and the CFO estimation of the k^{th} user is shown by V_k.

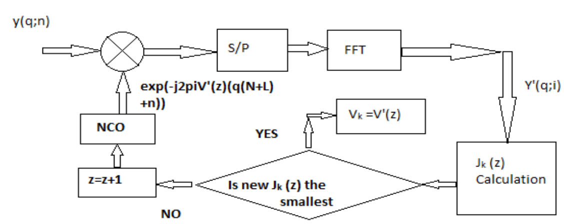

Fig. 6. Block diagram of CFO estimation block.

The estimation algorithm works as follows:

- Estimator block k is used for producing the estimation V_k. Each incoming block y(q;n) is multiplied with [exp(-j2piV'(z)(q(N+L)+n))] which is produced by numerically controlled oscillator (NCO) with a specific end goal to compensate the impact of CFO. Here q is the block index, L is the CP length and v'(z) is the guess of CFO in the z^{th} look for user k.

- The FFT block which is present in SC-FDMA technique works with a goal to change over the resulting sequence into frequency domain Y'(q;i).
- After repeating the first two steps for N_b blocks, $J_k(z)$ which is the average energy falling in the virtual subcarriers of the k^{th} user's sub-band is calculated.
- The first three steps are repeated for V'(z) sweeping the range [-1/2N,1/2N] in z steps (generally equivalent to the FFT size), until the V_k which provides the minimum $J_k(z)$ is found.
- V_k becomes the accurate CFO estimate and the blocks are reprocessed in the receiver with this new V'(z) and passed to the subcarrier demapping block and onwards.

For LFDMA with sub-bands, in the z^{th} search:

$$J_k(z) = \frac{1}{N_b}\sum_{q=0}^{N_b-1}\sum_{i=kJ_a+J+1}^{(k+1)J_a}\left|Y'(q;i)\right|\text{^}2 \quad \ldots(9)$$

is the average energy falling in the null subcarrier for N_b consecutive blocks. Also for IFDMA with sub-bands, in the z^{th} search:

$$J_k(z) = \frac{1}{N_b}\sum_{q=0}^{N_b-1}\sum_{i=0,i\neq kJ_0+nM_k}^{N-1}\left|Y'(q;i)\right|\text{^}2 \quad \ldots(10)$$

is the average energy falling in the null subcarriers for N_b consecutive blocks. The purpose behind attempting to find the estimate giving the minimum energy value is that if there was no noise, perfect synchronization would bring up null energy in the null subcarriers. Therefore, this forms the backbone of the CFO estimation strategy.

An additional block is required at the receiver side which is band-pass filters (BPF) block present before the CFO estimation process. This is vital for dispensing the impact of MAI in multiuser systems. Because, as each user has a distinctive CFO, even if the CFO of the user k is compensated totally (by multiplying y(q;n) by [exp(-j2piΔf_k(q(N+L)+i))] so that no energy of user k falls into its neighboring null subcarriers.

V. SIMULATION RESULTS

In this work, we consider a SC-FDMA system with parameters suitable for LTE uplink. Table 1 provides the parameters of SC-FDMA:

Table 1. Parameters of LTE used in SC-FDMA.

Channel bandwidth	5 KHz
Frame duration	10 msec
Sub-frame duration	1 msec
Sub-carrier spacing	15 KHz
FFT size	512
Number of data subcarriers	300
Modulation scheme	BPSK, QPSK
Cyclic prefix length	128
Mapping	LFDMA

In first simulation result, I have compared the results of PAPR of OFDM and SC-FDMA by using QPSK modulation techniques. This graph clearly shows that PAPR of SC-FDMA is low as compared to OFDM. When the probability is 10^{-4}, OFDM having PAPR around 12 dB but at the same point SC-FDMA having around 3dB PAPR.

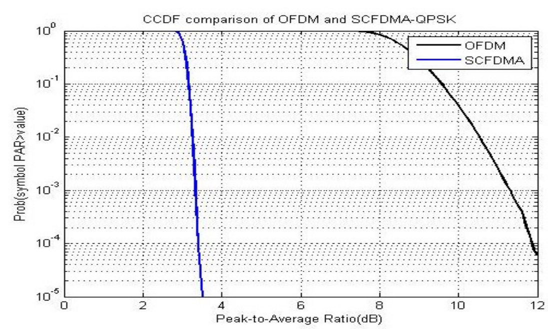

Fig. 7. CCDF comparison of PAPR 128pt FFT (QPSK).

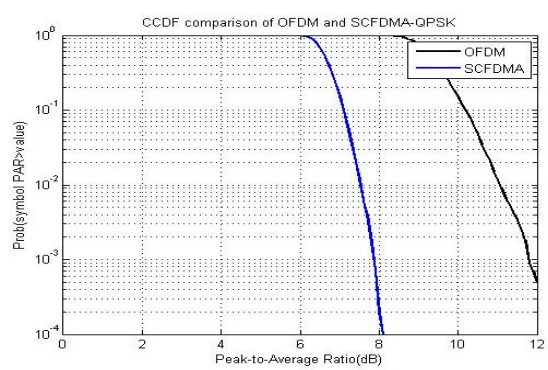

Fig. 8. CCDF comparison of PAPR 512pt FFT (QPSK).

In fig. 8, PAPR of OFDM and SCFDMA both having difference between their values.

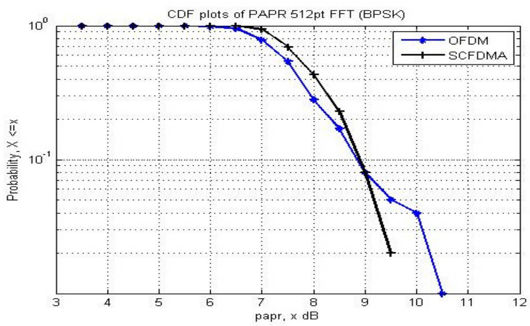

Fig. 9. CCDF comparison of PAPR 512pt FFT (BPSK)

When probability is 10^{-4} then the SCFDMA having PAPR 8 dB and OFDM having more than 12 dB. This happens because of changing the number of FFT points 512 pt. in place of 128 pt. and use different type of mapping.

In this plot, the PAPR of SC-FDMA is low at the bottom of the graph whereas OFDM having high PAPR. This plot originates because of 512pt. size of FFT and applying localized mapping with BPSK modulation scheme.

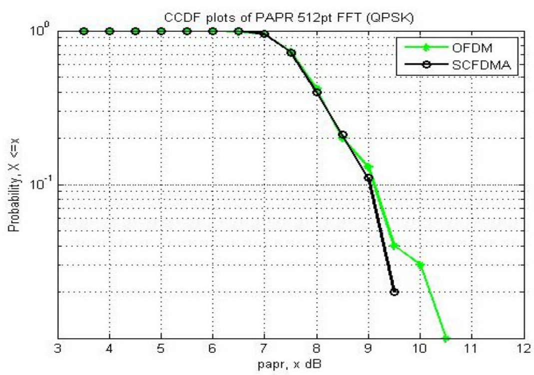

Fig. 10. CCDF comparison of PAPR 512pt FFT (QPSK).

This also shows the same results as we seen above that the SCFDMA having low PAPR as compared to SC-FDMA, because of having 512pt. FFT and localized mapping with QPSK modulation scheme.

From fig. 7, 8, 9 and 10, we only get that the PAPR of SC-FDMA is low with any modulation schemes and with any FFT points.

Fourth simulation, show the bit error rate (BER) and symbol error rate (SER) of SC-FDMA with respect to signal to noise ratio (SNR). In the fig. 11, BER and SER are very close to theoretical BER because there is no additional noise in the system.

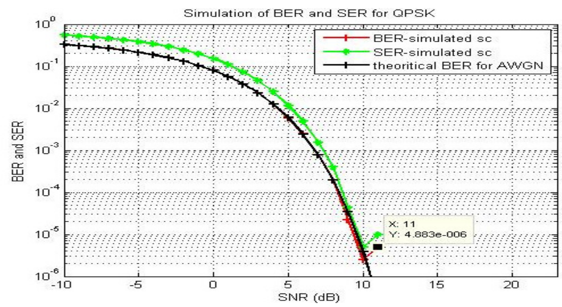

Fig. 11. BER performance of SC-FDMA.

In the second simulation, we provide power spectral density (PSD) graph, which describes how the power of the signal or time series is distributed with frequency. According to Wiener-Khinchin theorem, the PSD is the Fourier transform of Autocorrelation function of the signal, if the signal can be treated as a wide-sense stationary random process. The power of a signal in a given frequency band can be calculated by integrating over positive and negative frequencies. The power spectral density of a signal exists if and only if the signal is a wide-sense stationary process. If the signal is not stationary, then the autocorrelation function must be a function of two variables, so no PSD exists, but similar techniques may be used to estimate a time-varying spectral density.

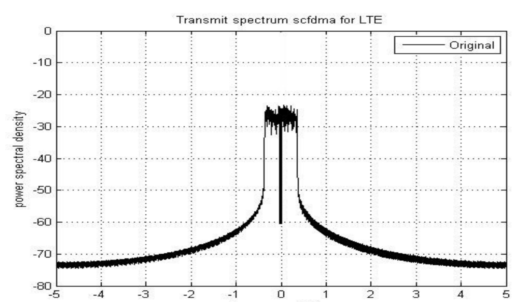

Fig. 12. Power spectral density of SC-FDMA.

In the third simulation results, number of subcarriers per symbol is 512 and number of subcarriers per user is 128. There are two users in the system and the subcarrier spacing is 15 kHz. For subcarrier mapping method, localized method is used. It can be seen from fig. 13 that the proposed CFO synchronization method highly decreases the error rate compared to the no CFO synchronization scenario. It also gives a close performance to the no CFO scenario.

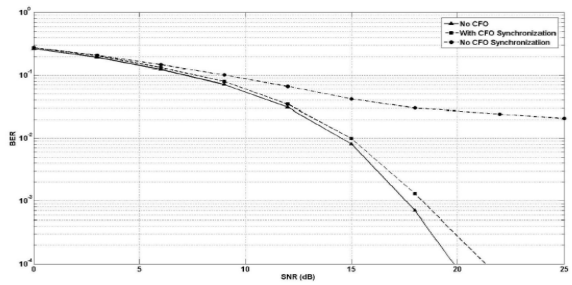

Fig. 13. BER performance of used method.

VI. CONCLUSION

In this paper, we have analyzed the system architecture of LTE uplink SC-FDMA systems, with some basic details of SC-FDMA. This paper also includes problems present in SC-FDMA which is ISI and CFO and also includes methods to remove them. We also show simulation results of comparison of PAPR of OFDM and SC-FDMA, BER and SER without any offset, PSD, and the effect of multiple CFOs, the null subcarrier method which is used to avoid this problem and obtained simulation results which show the effect of CFO synchronization method. In the first simulation, graph of theoretical and simulated of BER are almost at the same point. This shows the excellent performance of SC-FDMA. Next is the PSD graph which gives the power of signal with frequency. Third simulation shows that the proposed method highly increases the performance with respect to the non-synchronized scenario and brings it closer to the no CFO scenario.

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